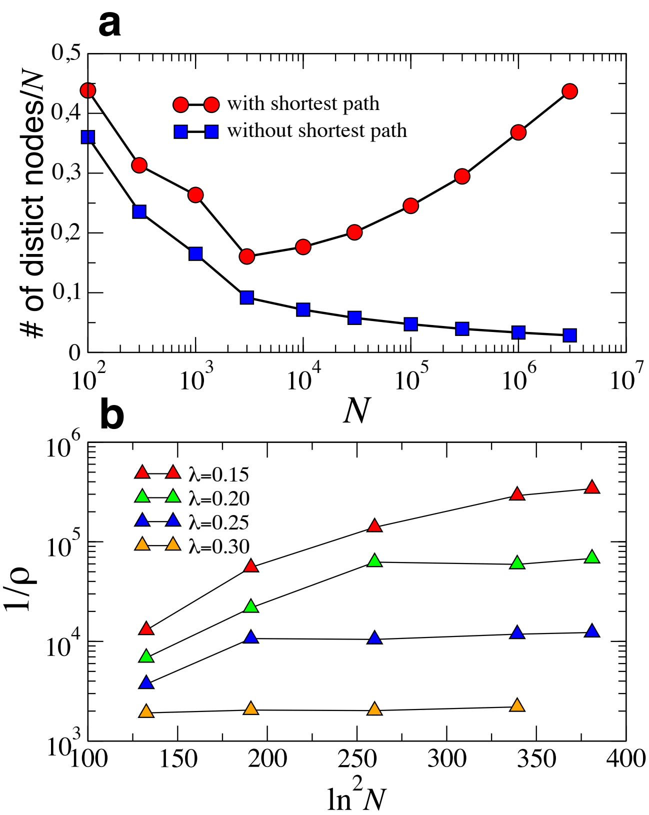

FIG. 1: **a.** Blue squares show the relative size of the subgraph made of nodes with $k > \ln N$ plus their direct neighbors as a function of N. As expected, this is a decreasing function of N. Red circles show the relative size of the same subgraph plus the number of distinct nodes connecting any pair of nodes with $k > \ln N$ with a shortest path. As it can be observed, this curve increases with N (and eventually converges), indicating that this subgraph is macroscopic. This happens even if we have only considered one shortest path per pair of nodes, so that the red circles are just a lower bound of the relative size of the real subgraph. **b.** Behavior of the inverse of the prevalence, as measured from the quasi-stationary method, as a function of the system size for a SF network with $\gamma = 4$ and $k_{min} = 2$. Values of the system size N range from 10^5 to 3×10^8. $\lambda_c^{HMF} = 0.328$, $\lambda_{max}(N = 10^5) = 0.3536$, $\lambda_{max}(N = 10^6) = 0.2878$,$\lambda_{max}(N = 10^7) = 0.2248$.

Boguñá *et al.* Reply: In their comment to our letter [1], Lee et al. [2] make an important point concerning the solution of Eq. (3) in [1]. They are right in pointing out that this equation predicts, for any given value of λ, epidemic activity only for nodes with degrees above $\ln N$. From this result, they conclude that the prevalence ρ is bounded by $(\ln N)^{-(\gamma-2)}$ and, thus, that it goes to zero in the thermodynamic limit. This observation is indeed correct as far as Eq. (3) is concerned. However, we would like to point out that Eq. (3) does not describe the original dynamics but an effective one that only considers infections among distant hubs mediated by chains of nodes. When translated back to the original dynamics in the real network, this subset of high degree nodes does lead to a truly endemic activity for any value of λ.

Indeed, in the modified dynamics, chains of nodes connecting high degree nodes are considered only as the way to propagate the infection among hubs but not part of the system per se. However, in the real dynamics, these nodes do actually belong to the network and, because they are the ones propagating the epidemics, they are necessarily active. The sum of nodes with degrees above $\ln N$ plus nodes connecting them is, in fact, a macroscopic fraction of the entire system, as we show in Fig. 1 a. As a consequence, above the threshold $\lambda_{max}(N)$ predicted by the modified dynamics, the activity in the original system is truly endemic.

To further check our results, we have made more numerical simulations with the quasi-stationary method in random networks with $\gamma = 4$ and $k_{min} = 2$. Figure 1 **b** shows the inverse of the prevalence $1/\rho$ in logarithmic scale as a function of $(\ln N)^{\gamma-2}$ for values of λ below the heterogeneous mean field (HMF) prediction λ_c^{HMF}. If the argument by Lee et al. were correct, we should find a linear increasing function for any value of $\lambda < \lambda_c^{HMF}$. Instead, we only observe a growing behavior for values of λ smaller than our prediction $\lambda_{max}(N)$, whereas we find a constant value when $\lambda_{max}(N) < \lambda < \lambda_c^{HMF}$.

We therefore conclude that the main result of our manuscript, namely the bound of the threshold of the SIS model as given implicitly by Eq. (4) in Ref.[1], is essentially correct, providing a reasonable theoretical estimate of its value.

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[1] M. Boguñá, C. Castellano, and R. Pastor-Satorras, Phys. Rev. Lett. **111**, 068701 (2013), URL `http://link.aps.org/doi/10.1103/PhysRevLett.111.068701`.
[2] H. K. Lee, P.-S. Shim, and J. D. Noh, arXiv:1309.5367.